PROBLEMS.

45. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find the first six sets of values in which the sum of two consecutive integral squares equals a square.

46. Proposed by B. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, Ohio.

If any positive integral number N be divided by another positive integral number D, leaving a remainder of 1, then any positive integral power of N, divided by D, will leave a remainder of 1.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

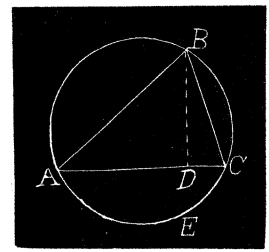
SOLUTIONS OF PROBLEMS.

28. Proposed by B. F. FINKEL, A. M., Professor of Mathematics, Drury College, Springfield, Missouri.

What is the average area of all triangles having a given base, b, and a given vertical angle, α?

Solution by the PROPOSER.

Let ABC be a triangle whose base $AC=b$ and vertical angle $ABC=\alpha$. Let $BC=x$, $\angle BAC=\theta$, and Δ average area required.



Then $x=\frac{b}{\sin\alpha}\sin\theta$, and $BD=x\sin\angle BCA$

$$=\frac{b}{\sin\alpha}\sin\theta\sin(\theta+\alpha).$$

$\therefore$ Area of the triangle $=\frac{b^2}{2\sin\alpha}\sin\theta\sin(\theta+\alpha)$.

The limits of θ are 0 and $\pi-\alpha$.

$$\therefore \Delta=\frac{\int_0^{\pi-\alpha}\frac{b^2}{2\sin\alpha}\sin\theta\sin(\theta+\alpha)d\theta}{\int_0^{\pi-\alpha}d\theta}=\frac{b^2}{2(\pi-\alpha)\sin\alpha}\int_0^{\pi-\alpha}\sin\theta\sin(\theta+\alpha)d\theta$$

$$=\frac{b^2}{2(\pi-\alpha)\sin\alpha}\left[(-\tfrac{1}{2}\sin\theta\cos\theta+\tfrac{1}{2}\theta(\cos\alpha+\tfrac{1}{2}\sin^2\theta\sin\alpha\right]_0^{\pi-\alpha}$$

$$=\frac{b^2}{4(\pi-\alpha)}\left\{(\pi-\alpha)\cot\alpha+1\right\}.$$